# A Study of Pilot Placement Optimization with Constrained MDPs in IEEE802.11p Systems

Yan Yang

State Key Lab. of Rail Traffic Control and Safety, Beijing Jiaotong University Beijing, China 100044 Email: yyang@bjtu.edu.cn Yejun He Shenzhen Key Lab. of Antenna and Propagation, Shenzhen University Shenzhen, China 518060 Email: heyejun@126.com Mohsen Guizani Electrical Engineering University of Idaho Moscow, ID, USA Email: mguizani@ieee.org

Abstract—This paper proposes a decision-assisted pilot placement optimization method in IEEE802.11p physical layer. The fast time-varying channel is first modeled as a typical Gaussian-Markov process. Under the constraint of state-spaces, the pilot optimization problem is further formulated as constrained Markov decision processes (MDPs). Secondly, for achieving compatibility with existing standards, our goal is to determine the optimal pilot placement and employ only very limited pilot patterns in response to fast varying channels. We develop a channel state matched pilot optimization method, where the optimization procedures focus on how to respond the different channel variations in the time and frequency domains. To jointly evaluate the severity of channel variations in the time and frequency domains, we derive an effective mutual information measurement criterion. Simulation and numerical results show the efficiency of the pilot optimization decision scheme in reducing the channel estimation error, and mutual information measurement can yield an accurate performance evaluation in relatively fast time-varying vehicular communication scenarios.

## I. INTRODUCTION

IEEE802.11p systems currently adopt the identical pilot pattern with the indoor IEEE 802.11a, a matured orthogonal frequency division multiplexing (OFDM) technology which mainly focuses on nomadic indoor usage. It has to face the underlying risks in Vehicle-to-Infrastructure (V2I) and Vehicle-to-Vehicle (V2V) communications owing to the delay spread and Doppler spread. Pilot placement has a significant impact on the high data rate transmission of pilot-assisted orthogonal frequency-division multiplexing (OFDM) systems.

There are basically two categories for the pilot placement optimization scheme. The first category focuses on finding the number of optimal pilot symbol patterns to maximize an upper bound for the constrained channel capacity. Several famous criteria have been well studied to assess optimal pilot patterns, for example, *mean squared error* (MSE), *bit error rate* (BER) and *symbol error rate* (SER) [2-8]. In [3], the authors exploited a heuristic algorithm to search for a

trustworthy pilot carrier placement with a low MSE, as well as an adaptive pilot placement. A summary of pilot pattern designs can be found in [4]. The second category seeks to develop a positive pilot design for channel estimation and carrier frequency offset estimation for inter carrier interference (ICI) cancellation. The concept of virtual pilots was first proposed in [9], and Budiarjo et al. further applied this concept to perform channel estimation of cognitve radio [7]. Recently, people have paid more attention to adaptive pilot placement. Šemiko [10] proposed a framework of adaptive pilot patterns, and attempted to obtain an upper bound of a constrained capacity. The contribution of this paper is the pilot optimization problem is formulated as constrained MDPs, and we provide a fresh insight on how to respond to the fast time-varying channel with very limited pilot patterns. In order to reduce the computational complexity associated to the pilot optimization procedure, we propose to track channel variations using a pilot patterns based on the MDPs. The proposed scheme for adaptive pilot placement is supported by reliable mathematical derivations and experiments, thus, it is applicable to fast-changing vehicular channel conditions.

The rest of the paper is organized as follows. In Section II, we describe the pilot optimization model. In Section III, we propose the mutual information assessment criterion and describe the detail optimization method based on constrained MDPs. An extensive simulation is presented in Section IV, and the paper concludes in Section V.

## **II. PROBLEM FORMULATION**

### A. OFDM Pilot Optimization Model

Considering the pilot-assisted OFDM system that employs N subcarriers, the time-frequency grid is equipped with  $N_p$  pilot subcarriers and  $N_d$  data subcarriers. For the kth transmitted data vector  $\mathbf{x}(k) = [x(0) \ x(1) \ \cdots \ x(k-1)]$ , it is transferred into the time domain transmitted data as a  $N \times N$  diagonal matrix  $\mathbf{s}(k) = \mathbf{diag}[s(0) \ s(1) \ \cdots \ s(k-1)]$  by an inverse discrete Fourier transform (IDFT), and the kth transmitted symbol can be written as

$$s(k) = \sum_{n=0}^{N-1} x(k) e^{j2\pi nk/N} \qquad k = 0, 1, ..., N-1.$$
(1)

<sup>&</sup>lt;sup>1</sup>This work was supported by the research task of the State Key Laboratory of Rail Traffic Control and Safety (RCS2017ZT010), the major research plan of China Railway Corporation (2016X009-A), the China Scholarship Council, the key research task of the Ministry of Education of the Peoples Republic of China (Grant No. K13C800010), the National Natural Science Foundation of China (No.61372077), the fundamental Research Progrogram of Shenzhen City (No. JC201005250067A/JCJY2012817163755061) and Guangdong Science and Technology Program (No. 2013B090200011).

Suppose  $\mathbf{s}(k)$  is transmitted over a *L*-tap multipath channel  $\mathbf{h}(k) = [h_1 \ h_2 \ ... \ h_L]^T$ , where  $h_L$  is the transfer function of the *L*th tap. Assuming that the discrete Fourier transform (DFT) at the receiver with coherent detection can be carried out correctly without ICI, and the cyclic prefix is removed. Then, the received data vector is demodulated by a unitary  $N \times N$  DFT matrix **W** 

$$\mathbf{y}(k) = \mathbf{s}(k)\mathbf{W}\mathbf{h}^{T}(k) + \mathbf{w}(k), \qquad (2)$$

where the matrix element of **W** is given by  $W_{nk} = e^{-j2\pi nk/N}$ and **w**(*k*) is an independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN).

Accordingly, considering that the additive ICI components only come from adjacent subcarriers, the *j*th subcarrier of the *k*th received signal in the frequency domain is represented as

$$Y_{j}(k) = H_{j}(k)S_{j}(k) + \underbrace{\sum_{i=-j, i\neq 0}^{N-1-j} H_{j+i}(k)S_{j+i}(k) + W(k)}_{\text{ICI}} \quad i \le |i_{\max}|,$$
(3)

where *i* is the index of the interference subcarrier,  $i_{max}$  is the maximum distance between the interference subcarriers and W(k) is noise. A simplified vector expression of the Eq. (3) can be written as

$$\mathbf{Y}(k) = \tilde{\mathbf{H}}(k)\mathbf{S}(k) + \mathbf{W},\tag{4}$$

where  $\hat{\mathbf{H}}(k)$  is the channel frequency response(CFR) matrix containing the multiplicative channel gain of the *k*th subchannel on its diagonal and the residual ICI gains on its non-diagonal entries, and **W** is the i.i.d. AWGN vector.

In general, the typical channel fading process can be modeled as a first-order Gauss-Markov process [4]. For IEEE802.11p systems, the radio link (e.g., V2V and V2I), should be characterized as time-varying and non-stationary [11]. Let  $h(\tau; t)$  and  $H(\tau; f)$  be the channel impulse response (CIR) and CFR in the time and frequency domains, respectively. Taking into account the effects of channel variance in the time and frequency domains, they can be modeled as

$$\mathbf{h}_n = \alpha \mathbf{h}_{n-1} + \mathbf{e}_{n-1} \tag{5a}$$

$$\mathbf{H}_k = \alpha \mathbf{H}_{k-1} + \mathbf{E}_{k-1},\tag{5b}$$

where *n* and *k* are indices,  $\mathbf{e}_{n-1}$ ,  $\mathbf{E}_{k-1}$  are zero-mean Gaussian random vectors that are independent of the noise, and  $\mathbf{H}_k$  is CFR. In addition, the coefficients  $\alpha \in (0, 1]$  shows how quickly the channel changes in the time and frequency domains respectively (Small  $\alpha$  corresponds to fast fading, and large coefficient refers to slow fading).

In this paper, we use parameter sets  $\chi$  and  $\psi$  to characterize the multipath fading channel in the time- and frequencydomain, respectively. Here,  $\chi : \{\alpha, \phi, \tau; t\}_{\ell=1}^{L}$  is defined as the channel variances in the time domain (amplitude:  $\alpha$ , phase:  $\phi$  and delay:  $\tau$ ). Similarly,  $\psi : \{A, \Phi, f_d; f\}_{\ell=1}^{L}$  is represented by the main parameters (amplitude: A, phase:  $\Phi$  and Doppler



Fig. 1. OFDM pilot optimization model based on MDPs

shift:  $f_d$ ) with respect to the frequency-domain. Then, the optimization problem can be formulated as

$$\max_{u \in \{\chi, \psi\}} C(u)$$
  
subject to  $\mathcal{D}^{\kappa}(\pi) < \mathcal{V}_{\kappa}, \quad \kappa = 1, 2, ..., \mathcal{K},$  (6)

where C(u) is the channel capacity,  $\mathcal{D}^{\kappa}(\pi)$  ( $\kappa = 0, 1, ..., \mathcal{K}$ ) is the cost criteria related to a policy  $\pi$ ,  $\kappa$  is a  $\mathcal{K}$ -dimensional vector of immediate costs, and  $\mathcal{V}_{\kappa}$  is the given performance bound [12]. Consequently, this optimization problem can be solved with a specified decision policy.

Based on the above understanding, we propose a novel OFDM pilot optimization scheme based on Markov decision processes (MDPs). MDPs also known as controlled Markov chains, constitute a basic framework for dynamically controlling systems that evolve in a stochastic way. A key Markovian property is that conditioned on the state and action at some time t, the past states and the next one are independent. MDPs can be defined by a tuple  $\mathcal{X}, \mathcal{A}, \mathcal{P}, c, d$ , where  $\mathcal{X}$  is a state space that contains a finite number of states, A is a finite set of actions,  $\mathcal{P}$  are the transition probabilities, c is an immediate cost and d is a K-dimensional vector of immediate costs. The main idea of this paper is that optimal pilot placement depends on the specific channel state, and adapts to channel state changes based on performance measurement. The optimum problem of pilot placement is formulated as a constrained MDP, where the steady decision for pilot placement selection acts not only as the channel varying in the time domain but also in response to the frequency domain. Fig. 1 illustrates the pilot optimization model, where the optimal pilot placement is chosen by a Markov decision process  $\mathcal{M}$  to provide high transmission performance.  $T_s$  is the period of a pilot placement in the time grid, i.e., the length of the smallest block over which the placement pattern repeats.  $f_s$  is the normalized pilot symbol spacing in the frequency grid, i.e., the number of the allocated pilot subcarriers. As a pilot can be viewed as a sounding probe signal or channel sampling, specific pilot pattern could be the result of the regular periodic placement of pilot symbols, denoted by  $f_s$  and  $T_s$ . For a given placement  $\mathcal{P}$ , a binary string can be used to represent the the placement pattern of pilot symbols in the frequency domain, where 0 and 1 represents information symbol and pilot, respectively.  $\mathscr{P}$  can be denoted as a 3-tuple  $\mathscr{P}(m, \Delta, \mathbb{Z})$ 

$$\mathscr{P}(m,\Delta,\mathbf{Z}) = \begin{cases} \sum_{i=0}^{N_P} 2^{\mathbf{Z}+i\times\Delta(t)} & m = nT_s \\ 0 & m \neq nT_s, \end{cases}$$
(7)

where *m* is the period of insertion of pilot symbols,  $\Delta$  is the interval of pilot subcarriers,  $N_P$  is the number of pilots and Z is the first pilot subcarrer location at the time-frequency grid.

In Fig. 1, we consider a constrained Markov decision process  $\mathcal{M}$  with finite state, action, and reward spaces, thus, the computation of (6) appears to have lower complexity. In this case,  $\mathcal{M}$  is employed to determine the optimal  $\mathscr{P}^*(m, \Delta, \mathbb{Z})$ , which aims to deal with the different channel conditions. Typically,  $\mathcal{M}$  is a 5-tuple  $\mathcal{M} = (\mathcal{S}, \mathcal{A}, \mathcal{A}(\vec{s}), \mathcal{T}, \mathcal{R})$ , where  $S : \{\vec{s_1}, \vec{s_2}, ..., \vec{s_n}\}$  is a state space that contains a finite number of states,  $\mathcal{A} : \{a_1, a_2, ..., a_n\}$  is a set of actions,  $\mathcal{A}(\vec{s})$  is the set of admissible actions in state  $\vec{s}$ ,  $\mathcal{T}$ : [0,1] is the transition probability function and  $\mathcal{R}$ :  $(\vec{s}, a)$  is the total reward function [13-14]. Here, the state set S is assumed to be associated with the varying channels  $\mathbf{h}_n : {\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_n}$ . Once the MDPs reach a steady state  $\vec{s}_i, \vec{s}_i \in S$ , we should input a strategy that the chosen action  $a_i \in A$  is to obtain the optimal  $\mathscr{P}^*(m, \Delta, \mathbb{Z})$  in response to the probability distribution  $\mathbf{h}_i$ . For any policy  $\pi$  and initial state distribution  $\beta$ , the cost function  $C^n(\beta, \pi) = \sum_{t=1}^n E_{\beta}^{\pi} c(\mathcal{S}_t, \mathcal{A}_t)$  can be used in applications of control of MDPs [14].

To validate the performance of pilot pattern, this paper adopts a minimum mean-square error (MMSE) estimator, and the optimization problem of (6) can be rewritten as

$$\max_{\substack{u \in \{\chi,\psi\}}} C(u) \equiv \min_{\substack{h \in S}} \mathbb{E}\left[\left|h - \hat{h}_{p}\right|^{2}\right] \quad \forall S$$
  
subject to  $\mathcal{D}^{\kappa}(\pi) < \mathcal{V}_{\kappa}, \quad \kappa = 1, 2, ..., \mathcal{K}$   
and  $\mathscr{P}^{*}(m, \Delta, Z),$  (8)

where  $\hat{h}_p$  is the estimates of CIR at the pilot position. Traditionally, (8) can be solved by the exhaustive search, but the complexity is always high [3].

## B. Complexity Trade-offs

We believe that many more pilot patterns might not be better. In fact, it is impossible to find all possible channel matched pilot patterns because the number of state-spaces S is far greater than the number of pilot patterns  $\mathcal{P}$ . Although more pilot patterns might achieve a higher network throughput, it is unattractive for practical applications since the computation of an optimal policy is very costly (e.g., exhaustive searching). For the channels exhibiting limited states, consequently, we consider a pilot optimization scheme based on constrained state-space. The basic idea is to generate limited pilot patterns, and choose a near-optima pilot placement to deal with the fast varying channels. With respect to channel estimation in fast time-varying vehicular channels, we develop an enhanced pilot scheme to improve the estimation performance. We further propose three enhanced pilot patterns, where pattern 0 represents fixed pilot placement, pattern 1 and 2 represent enhanced

pilot placement. In pattern 0, four pilot subcarriers are added by taking the values 0 4 001 0008 0020, where time and frequency domain representation of pilots with inserted rate  $T_s$  and  $f_s$ . In pattern 1, the inserted rate is 1/2Ts and  $2 \times f_s$ , and eight pilot subcarriers are added by taking the values 0 4081 0208 1021. In pattern 2, the inserted rate is the same as that in pattern 0, and the difference is that four pilot subcarriers are added by taking the alternated values 0 4 001 0008 0020 and 0 0080 0200 1001, respectively. Generally, there are two major issues in pilot placement: (1) the channel capacity/ spectral efficiency, and (2) the capacity/ spectral efficiency is associated with pilot placement (e.g., the percentage of pilot symbols in the data stream). It is clear that the enhanced patterns do not change the density of pilots on the time-frequency grid, thus, our pilot placement could be utilized without any capacity loss. By using a virtual pilot-aided channel estimation method, additional pilots can be exploited to yield accurate channel estimates and track carrier frequency offset (the detail of virtual pilots has been well studied in [7][9]).

#### **III. PILOT PLACEMENT OPTIMIZATION**

#### A. Mutual Information Measurement Criteria

From the received signal model in (1) and (2), the mutual information in time and frequency domain are given by

$$I(x; y) = \mathcal{H}(x) - \mathcal{H}(x|y)$$
(9a)

$$I(X;Y) = \mathcal{H}(X) - \mathcal{H}(X|Y), \tag{9b}$$

where  $\mathcal{H}(\cdot)$  denotes entropy. Similarly, let  $x = \{x^1, x^2, ..., x^i\}$ and  $X = \{X^1, X^2, ..., X^i\}$  be the transmitted signals in time and frequency domain, accordingly,  $y = \{y^1, y^2, ..., y^j\}$  and  $Y = \{Y^1, Y^2, ..., Y^j\}$  are the received signals. We can calculate the average mutual information using

$$I(x; y) = \mathbb{E}\left[I(x^{i}; y^{j})\right] = \sum_{i} \sum_{j} p(x^{i}, y^{j}) \log_{2} \frac{p(x^{i}, y^{j})}{p(x^{i})p(y^{j})}$$
(10a)  
$$I(X; Y) = \mathbb{E}\left[I(X^{i}; Y^{j})\right] = \sum_{i} \sum_{j} p(X^{i}, Y^{j}) \log_{2} \frac{p(X^{i}, Y^{j})}{p(X^{i})p(Y^{j})},$$
(10b)

where  $p(\cdot)$  is used to describe the posterior distribution in time- and frequency-domain, respectively. Mutual information has been verified as an efficiency criterion for performance assessment and is equivalent to the traditional estimation theory (e.g., least square (LS) and MMSE) [11].

Consequently, for pre-defined channel variant collection  $\chi$  and  $\psi$ , we can jointly assess the effect of channel parameters  $\phi$  and  $\Phi$  in the time- and frequency-domain, which are given by

$$\frac{\partial}{\partial \phi} I(x|y) = \mathbb{E}\left[\frac{\partial \log_2 P^{\phi}_{y|x}}{\partial \phi} \log_2 P^{\phi}_{x|y}\right], \phi \in \chi$$
(11a)

$$\frac{\partial}{\partial \Phi} I(X|Y) = \mathbb{E}\left[\frac{\partial \log_2 P^{\Phi}_{Y|X}}{\partial \Phi} log_2 P^{\Phi}_{X|Y}\right], \Phi \in \psi.$$
(11b)

Observing (11a) and (11b), we can find a distribution that it can maximize I(X; Y) and I(x; y), while also maximizing the channel capacity. We determine  $\mathcal{P}^*$  numerically by maximizing the mutual information with binary inputs. Hence, the joint optimization problem is to maximize capacity C in the time- and frequency-domain. For pilot-assisted OFDM system, it can be represented as

$$\arg \max_{\phi \in \chi} C\left(\phi : x; y | \hat{h}_{p}\right) or \arg \max_{\Phi \in \psi} C\left(\Phi : X; Y | \hat{H}_{p}\right)$$
  
subject to  $\mathscr{P}^{*}(m, \Delta, Z),$  (12)

where  $\hat{H}_p$  is the estimation of CFR at the pilot positions. The next section will develop a steady policy to obtain  $\mathscr{P}^*$ .

#### B. Constrained Markov Decision Process

Denote a specified MDP associated with the time-varying wireless channels as  $\mathcal{M} = (S, \mathcal{A}, \mathcal{A}(\vec{s}), \mathcal{T}, \mathcal{R})$  and define  $\mathbb{Z}(S, a)$  as the evaluation of channel variation. Notice that  $\mathbb{Z}(S, a)$  can be clearly evaluated using mutual information criteria. Suppose we have an action at different time epochs  $a_j \in \mathcal{A}$  with expected reward  $\mathcal{R}$ . For any state  $\vec{s_i}, \vec{s_j} \in S$ , define  $\mathbb{Z}(\vec{s_i}, a_i)$  as the estimate of channel variation, if  $|\mathbb{Z}(\vec{s_i}, a_i) - \mathbb{Z}(\vec{s_j}, a_j)| < \varepsilon$ , then  $\vec{s_i}, \vec{s_j}$  can be aggregated to the k subset  $\mathcal{S}_k$ , it is also true that  $\mathcal{S}_k$  is a subset of S, i.e.,  $\mathcal{S}_k \subseteq S$ .

Consequently, under the state-spaces constraint, we can significantly reduce the number of state-spaces and easily solve value iteration using linear programming, and the action sets can further be reduced. In the previous discussion, we have shown that typical channel states can be summarized as the channel variations in the time- and frequency-domain. Furthermore, the channel states can be simplified as a series of subsets, e.g.,  $S^t$ ,  $S^f$  and  $S^{t-f}$ , which represent the three typical intensities of channels variations. At the beginning, we assigned a fixed 802.11p pilot pattern to  $S_1^t$  in the absence of any measurement information.

Let the expected reward for taking the action be policy  $\pi$ , which aims to obtain  $\mathscr{P}^*$ . A suboptimum decision procedure for  $\mathscr{P}^*$  can be presented as four steps.

**Step 1:** We use  $\mathcal{V}^{\pi}$  to denote the value function for policy  $\pi$ , and the state space is limited as a subset  $\mathcal{S}_i$ , e.,g.,  $\mathcal{S}_i^t$ ,  $\mathcal{S}_i^{f}$ ,  $\mathcal{S}_i^{t-f}$ 

$$\mathcal{V}^{\pi}(\mathcal{S}_i) = \mathcal{R}(\mathcal{S}_i) + \gamma \sum_{\mathcal{S}'_i} \mathcal{T}(\mathcal{S}_i | \mathcal{S}_i, a = \pi(\mathcal{S}_i)) \mathcal{V}_{\pi} \mathcal{S}'_i, \qquad (13)$$

where  $\gamma$  is the discount factor  $\in (0, 1]$ .

**Step 2:** According to the Bellman equation [12], the optimal value function  $\mathcal{V}^*$  can be iterated by using

$$\mathcal{V}^{*}(\mathcal{S}_{i}) = \max_{a \in \mathcal{A}} \left\{ \mathcal{R}(\mathcal{S}_{i}, a) + \gamma \sum_{\mathcal{S}'_{i}} \mathcal{T}(\mathcal{S}'_{i} | \mathcal{S}_{i}, a = \pi(\mathcal{S}_{i})) \mathcal{V}^{*}_{\pi}(\mathcal{S}'_{i}) \right\},$$
(14)

where  $\mathcal{V}^*_{\pi}(\mathcal{S})$ ,  $\vec{s} \in \mathcal{S}_i$ , is unique.

**Step 3:** The optimal policy  $\pi^*$  for state transition is

$$\pi^{*}(a) = \arg \max_{a \in \mathcal{A}} \left\{ \mathcal{R}(\mathcal{S}_{i}, a) + \gamma \sum_{\mathcal{S}'_{i}} \mathcal{T}(\mathcal{S}'_{i} | \mathcal{S}_{i}, a = \pi(\mathcal{S}_{i})) \mathcal{V}_{\pi}^{*}(\mathcal{S}'_{i}) \right\}.$$
(15)

**Step 4:** According to (8), the output  $\mathscr{P}^*$  based on MMSE estimators is used for optimal channel estimates. Futhermore, the channel estimates at the virtual pilot positions can be predicted from past observations by cure fitting or linear interpolating, and the obtained estimates are regarded as approximate estimates of pilot [4] [7].

In steps 1-3, we have noticed  $\mathcal{R}(S_i, a) \equiv 1/I(X; Y)$ . In Section II, we define a 3-state Markov model as an input to the MDPs, so the complexity of the value iteration algorithm is low. Next, a linear programming algorithm can be carried out until the value of  $\mathcal{V}^*(S_i)$  is determined [12-13].

#### **IV. NUMERICAL RESULTS**

## A. Simulation Settings

The performance of the proposed pilot placement optimization with constrained MDPs has been evaluated by extensive simulation. The OFDM signal used for 802.11p comprises 52 subcarriers. Of these 48 are used for the data transmission and four are sued as pilot subcarriers. The separation between the individual subcarriers is 0.3125 MHz. This results from the fact that the 20 MHz bandwidth is divided by 64. As aforementioned in Sectiion II, the indices of the pilot subcarrier can be computed from (7). The physical-layer parameter settings conform to 802.11p standards (typical simulation settings: 52 subcarriers, QPSK modulation and 1/2 coding rate). We adopt the fast time-varying channel estimation scheme in [15] and the V2V channel model in [11]. In our simulation, the basic extension model (BEM) is utilized to accurately approximate the time-varying channel by a weighted sum of complex exponentials  $h(n,k) = \sum_{q=1}^{Q} h_q(k)e^{j\omega_q k}$ , where the coefficients  $h_q$ and frequency  $\omega_q$  are the estimated parameters. Owing to the highest energy efficient and perfect frequency band-limited, the discrete prolate spheroidal BEM (DPS-BEM) sequences can be used to parameterize the deterministic channel (due to space restrictions, the readers can refer to the detail in above literatures). We consider three typical vehicle scenarios, highway (120, 90 km/h) and urban (50 km/h).

### B. Performance Analysis

Figs. 2 and 3 illustrate the measurement performance of the average mutual information criteria in the time- and frequency domain, and we evaluate the ICI due to loss of orthogonality (by time delay and frequency offset). Note that the channel codec is not considered in baseband processing, and the resolutions would be significantly increased in high SNR conditions (e.g., 20 dB). We selected the RMS delay spread of a TU6 channel  $\tau_{rms} = 1.1 \,\mu s$  to reveal the characteristic of channel in the time domain, while setting the delay between two channels as  $\tau_1 = 30 ns$ ,  $\tau_2 = 300 ns$ ,  $\tau_3 = 900 ns$ . Fig. 2 shows that with the increase of SNR, the resolution increases accordingly. Fig. 3 shows the evaluation of the channel variation in the frequency domain. According to the equation (3), we only consider the additive ICI components. Another observation from Fig. 3 is that there is no significant improvement of capacity in high speed conditions, where ICI



Fig. 2. Average mutual information vs SNR: I(x; y) were obtained from 1000 OFDM symbols.  $\alpha$  is set as 0.95.



Fig. 3. Average mutual information vs. SNR: I(X; Y) were obtained from 1000 OFDM symbols.  $\alpha$  is set as 0.90.

gain is set as  $\delta_1 = 3 \text{ dB} (50 \text{ } \text{km/h})$ ,  $\delta_2 = 5 \text{ dB} (90 \text{ } \text{km/h})$  and  $\delta_3 = 10 \text{ dB} (120 \text{ } \text{km/h})$ . To accelerate the value iteration, we used  $\lambda = 0.985$  for the discount factor with  $\varepsilon = 0.015$  for convergence. It ran for 1000 iterations before it converged.

Fig. 4 shows the MMSE channel estimator performance, where the enhanced pilot patterns are employed (refers to Fig. 2). We adopt a linear fitting method to obtain the predicted CIR at the virtual pilot position. We observe that there are significant performance gains through introducing virtual pilots. Fig. 4 also investigates two different high mobility scenarios, and one can see that the MSE is low which suggests a robust estimator performance.

## V. CONCLUSION

In this paper, we have proposed a low-complexity pilot placement optimization method in IEEE802.11p physical layer based on constrained Markov decision processes. A channel state matched pilot optimization method has been developed. We have derived an efficient mutual information criteria to evaluate the characteristics of the channel varying in the time



Fig. 4. Channel estimator performance: MSE vs SNR, OFDM block length N = 2000,  $N_P = 4$ .

and frequency domains. The main advantage of this scheme is that very limited pilot placements can be employed in response to fast varying channels, and may be easily extended to the other pilot-assisted OFDM and MIMO-OFDM systems.

#### References

- S. Coleri, et.al. Channel estimation techniques based on pilot arrangement in OFDM systems, *IEEE Trans. Broadcast.*, vol. 48, no. 3, pp.223-229, Sep. 2002.
- [2] X. Cai and G. B. Giannakis, Error probability minimizing pilots for OFDM with M-PSK modulation over Rayleigh-fading channels, *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 146155, Jan. 2004.
- [3] H. Steendam, How to Select the Pilot Carrier Positions in CP-OFDM?, in IEEE ICC, pp.3148-3153, 2013.
- [4] L. Tong, B. Sadler, and M. Dong, Pilot-Assisted Wireless Transmissions: General Model, Design Criteria, and Signal Processing," *IEEE Signal Processing Magazine*, vol. 21, no. 6, pp. 12-25, Nov. 2004.
- [5] T. Cui, and C. Tellambura, "Joint Frequency Offset and Channel Estimation for OFDM Systems Using Pilot Symbols and Virtual Carriers," *IEEE Trans. on Wireless Commun.*, vol. 6, no. 4, pp.1193-1202, Apr. 2007.
- [6] X. Ma, M. Oh, G. B. Giannakis, and D. Park, Hopping Pilots for Estimation of Frequency-Offset and Multiantenna Channels in MIMO-OFDM," *IEEE Trans on Comm.*, vol. 53, no. 1, pp.162-172, Jan. 2005.
- [7] I. Budiarjo, *et al.*, On The Use of Virtual Pilots with Decision Directed Method in OFDM Based Cognitive Radio Channel Estimation Using 2x1-D Wiener Filter", *ICC*'2008, pp. 703-707, 2008
- [8] J. Baek and J. Seo, Efficient Pilot Patterns and Channel Estimations for MIMO-OFDM Systems," Vehicular Technology, *IEEE Trans on Broadcast.*, vol. 58, no. 4, pp. 648-653, Dec. 2012.
- [9] P. Banelli, R. C. Cannizzaro and L. Rugini, "Data-Aided Kalman Tracking for Channel Estimation in Doppler-Affected OFDM Systems," *in IEEE ICASSP*, 2007.
- [10] M. Šimko, P. S. R. Diniz, Q. Wang, and M. Rupp, Adaptive Pilot-Symbol Patterns for MIMO OFDM Systems," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 9, pp.4705-4715 Sep. 2013.
- [11] P. Paschalidis, et al., Pathloss and multipath power decay of the wideband car-to-car channel at 5.7 GHz, in IEEE VTC Spring, 2011.
- [12] D. Guo, S. Shamai, and S. Verdú, Mutual information and minimum mean-square error in Gaussian channels, *IEEE Trans. Inform. Theory*, vol. 51, no. 4, pp. 12611282, April 2005.
- [13] E. A. Feinberg, A. Shwartz (eds.), Handbook of Markov decision processes: methods and applications, Springer, 2002.
- [14] E. Atlman, Constrained Markov decision processes. CRC Press, 1999.
- [15] Z. Tang, R. C. Cannizzaro, G. Leus, and P. Banelli, Pilot-assisted time-varying channel estimation for OFDM systems, *IEEE Trans.Signal Process.*, vol. 55, no. 5, pp. 2226-2238, May 2007.